Journal Academica Vol. 10(1), pp. 3-10, December 31 2021 - Education - ISSN 2161-3338

Full Research Paper

How do Teachers Define Infinity? Process or Object?

Layla Nasr*, Nina Hayfa
Lebanese University, Faculty of Education (Lebanon)

Received February 8 2021; Accepted March 24 2021

ABSTRACT

The present study investigates the conceptions of some teachers of mathematical infinity. A group of 32 pre-service and in-service teachers were asked to define the concept of infinity in mathematics. Teachers' definitions were analyzed and categorized according to the APOS theory. Results showed that more than half of the participants' definitions indicated a mixed conception of process and object at the same time.

Keywords: infinity, APOS theory, concept, conception, misconception

INTRODUCTION

It is known that the human mind has encountered many obstacles throughout history in the development of mathematical concepts, especially in contemplating abstract concepts like infinity. Cornu (1991) considered the notion of the infinitely large and the infinitely small as one of the major epistemological obstacles of the past. (as cited in Moru, 2006)

According to Tall (1999), infinity causes various problems to learners due to the duality in its meaning, as a process and as a concept. Nevertheless, not only students hold misconceptions of infinity. Previous research reports the existence of such misconceptions in pre-service teachers, in-service teachers, and even PhD students (Aztekin & al., 2010, Schwarzenberger and Tall, 1978, Tall, 1980, Dubinsky, 2013, Kattou, 2010). Teachers' conceptions of the infinite are therefore reflected to their students. This is because teachers are mediators between the knowledge to be learned and the students. Accordingly, their conceptions are of great importance as well as the terminology they use in expressing abstract concepts such as infinity. According to Tall (1990), "the message may be framed in a language that evokes inappropriate ideas and may be present in a sequence that is inappropriate for cognitive development.".

This paper is a part of a wider study that included investigation of students' conceptions in some Lebanese official schools as well. Results showed that students hold numerous misconceptions of infinity. Therefore, checking teachers' conceptions and the terminology used by them when addressing this concept is of great importance.

This study raises the following questions:

• How do teachers define the mathematical infinity?
• Do their definitions indicate a process or object conception of infinity?

*Corresponding author: laylanasr1@hotmail.com

THEORETICAL BACKGROUND

The APOS Theory:

The APOS theory is a constructive theory of learning based on Piaget's theory of reflective abstraction and applied to learning mathematical concepts for undergraduate level (Dubinsky & McDonald, 2001).

The acronym APOS, denoting Action, Process, Object, Schema, refers to the types of mental structures an individual build in responding to certain problem-solving situations. An individual uses certain mental mechanisms, such as interiorization, coordination, and encapsulation to construct these structures (Dubinsky et al., 2008).

According to the APOS theory, the formation of a mathematical concept begins by applying transformations on existing mental objects. The transformation which is triggered by memory or step-by-step instruction is termed as action. Upon repeating and reflecting on an action, this action could be interiorized to a process (Dubinsky & McDonald, 2001).

A process is a mental structure that performs transformation of objects but in the mind of an individual without executing each step explicitly. When an individual becomes aware of the process as a totality and realizes that transformations can be applied to it and is able to construct such transformations, this individual is said to have encapsulated the process into a cognitive object (Dubinsky & McDonald, 2001).

The collection of actions, processes, objects, and other schemas form the schema of a certain mathematical concept. Schemas are considered a framework which is used in a problem situation involving that concept (Dubinsky & McDonald, 2001).

The term genetic decomposition refers, according to the theory, to detailed descriptions of the mental constructions which students might use in their understanding of a certain mathematical topic. (Dubinsky & McDonald, 2001).

LITERATURE REVIEW

Revising literature, one could easily find how much the concept of infinity is conflicting and hard to grasp for students and even teachers. Fischbein (in Dubinsky, 2005) argued in reference to actual infinite sets that it is "contradictory in natural intuitive terms". Tall (2001b) considered that the concept of infinity arises from reflecting on finite experiences and extending them to the infinite. The extrapolation of various properties that are applicable in finite cases, give an intuition of infinity which leads to conflictions.

Many are the studies that assert the role of teachers and teaching strategies in the construction of the infinity concept in students' minds. Specifically, these studies address teachers' conceptions and the language used by them in expressing these conceptions.

Handal (2003) studied teachers' mathematical beliefs in instruction. The paper argues that these beliefs originate from previous traditional learning experienced mainly during schooling and are reflected in their instructional practices. And so, the misconceptions will pass from one generation to another if no steps to be undertaken for reforming teachers' conceptions. A study on student-teachers' understanding of infinity in a geometrical context revealed that the discussion about infinity could lead to the development of cognitive ability and pointed out the importance for teachers to have correct knowledge of infinity and to have communication skills necessary to reach the students' minds. (Jirotkova, Littler, 2004)

Previous literature shows the importance of presenting the concept in different contexts. Tall (2001a) suggests providing richer experiences for students involving a balance between the variety of examples and non-examples so that students are able to build a coherent concept. Voica & Singer (2003) found out that arguments of students were consistent, when there were connections between algebraic and geometric thinking.

A study done by Dubinsky and his colleagues (2013) on two groups of pre-service teachers' conceptions regarding the equality, one of which received APOS-based instructions and the group received traditional instruction showed a more stable belief for the group who received the APOS-based instruction.

METHODOLOGY

Thirty-two master students in Mathematics education in the Lebanese University were chosen for this study. All of them are mathematics teachers most of which are in-service. All the participants are BS holders in pure or applied mathematics. The participants were asked to "define infinity in mathematics."

The reason behind formulating this task was to keep the participants free in choosing their own approach and terminology and to unfold their conceptions of infinity without any restrictions or given directions.

This questionnaire was held in the academic year 2013/2014, in the Lebanese University and lasted for 10 minutes. The participants were not informed previously about the questionnaire. Participants were told that this questionnaire was for research sake and they were free not to write their names. Each participant worked individually.

Gathered data were collected and analyzed qualitatively according to the genetic decomposition of the APOS theory.

The data was classified into three categories:

- Indication of process
- Indication of object
- Indication of both process and object

In the third category, "indication of process and object" is the case where one phrase indicates process thinking and another indicates the object thinking for the same participant.

The classification of the sentences will follow the genetic decomposition of infinity according to APOS.

The participants' papers are to be coded as *E#* or *F#*, where *E* refers to English section and *F* refers to the French section and the number after refers to the student's number.

Segments of the answers given are written next to each participant's number as an evidence of the classification.

RESULTS AND CONCLUSION

Table 1: Participant's code with the indication of process

Participant's code	Indication
E3	… has no end… …unreachable… …limit…
E4	…unlimited… …tends to…
E6	…numbers have no end…
E13	…limiting concept… …cannot be attained… …estimation explanatory technique…
E14	…limit… …as time goes forever…
F1	…tends towards…
F5	…can't limit numbers…

Table 2: Participant's code with the indication of object

Participant's code	Indication
E1	…Number…
E2	…Inaccurate number…
E9	…Concept…
E12	…not defined number…
E15	$\mathbb{R}=(-\infty,+\infty)$
F2	…number beyond our conception…
F3	…undetermined entity
F6	…Number…
F7	…Very large number…
F9	…Number not precised…
F10	…Very large number…

Table 3: Participant's code with indication of both Process and Object

Participant's code	Indication of Process	Indication of Object
E5	Never have an end	Quantity
E7	$\lim_{x \to 0} \frac{1}{x} \to \infty$	Ratio
E8	…We can't reach…	Very large number
E10	Limit of unbounded increasing sequence	-Very, very, very big number -Value of 1/0
E11	Can't be measured	-Very large quantity -Indefinite quantity
F4	unlimited	Précised value
F8	…and unattainable at the same time	Largest number attainable
F11	unattainable	Largest unknown number
F12	Unknown and unattainable	Very large number
F13	-Unattainable -tends to a number	Very large number
F14	Unattainable in $\mathbb{R}$	Huge number
F15	-Intersection of two parallel lines -unattainable	-number -quotient of a number by zero
F16	-greatest which cannot be reached. -unattainable	it is the limit to infinite
F17	-unlimited -doesn't exist in reality	-number

As the tables above show, only 6 out of 32 used terminology that indicates thinking of infinity as a process, 12 out of 32 used words that indicate object, whereas, 14 out of 32 used words that indicate process thinking and words that indicate object thinking of infinity at the same time. This result shows that:

- It is hard, even for experts in mathematics, to define infinity. If not, infinity would have had an explicit definition in mathematics. This result is compatible with Tall (1990) "The mathematics contains concepts such as limit and infinity, which carry complex meanings that may be interpreted in inconsistent ways".
- Around half of the participants had mixed answers (process and object). This might be because they have experienced infinity in many contexts, some of which trigger the process thinking and other of which trigger the object thinking of infinity.
- Out of 32, 12 used terminology that indicates object approach for infinity. On the other hand, only 6 gave definitions that indicate process thinking. According to Tall (1981), some university students are more likely to believe in the actual infinity, whereas school students tend to believe in potential infinity. *Tall* considers that these beliefs go back to the kinds of experiences an individual has with infinity (Tall, 1981).

Concluding, it is hard even for teachers to have a coherent and clear conception of infinity. It seems that the struggle between the process and object conception of infinity in the human mind is never to end. Results of the study are compatible with results of previous research done on the subject (Kattou, 2010, Tall, 1980., Dubinsky et al., 2008&2013).
The concept of infinity is of great importance in mathematics, as it is integrated in many branches of it. Hence, future studies on this subject with larger populations should be conducted as well as studies that consider guiding and reforming teachers' conceptions of infinity.

REFERENCES

Aztekin, S., Arikan A., Sriraman, B. (2010). The constructs of PhD students about infinity: An Application of Repertory Grids. The Montana Mathematics Enthusiast, ISSN 1551-3440, vol.7, no.1, pp.149-174.

Dubinsky, E., Arnon, I., Weller, K. (2013). Pre-service Teachers' Understanding of the Relation Between a Fraction or Integer and its Decimal Expansion: The Case of 0.9 and 1. Canadian Journal of Science, Mathematics and Technology Education, 13(3), 232–258, 2013.

Dubinsky, E., McDonald, M.A. (2001) APOS: A constructivist Theory of Learning in Undergraduate Mathematics Education Research. Retrieved from: http://www.math.wisc.edu/~wilson/Courses/Math903/ICMIPAPE.PDF

Dubinsky, E., Stenger, C., Vidakovic, D. (2008). Infinite Iterative Processes: The tennis Ball Problem, European Journal of Applied Mathematics, 1-23.

Dubinsky, E., Weller, K., McDonald, M.A., Brown, A. (2005). Some historical issues and paradoxes regarding the concept of infinity: an APOS-based analysis: Part1.Educational Studies in Mathematics, 58, 335–359.

Handal, B. (2003). Teachers' mathematical beliefs: A review. Published in: The Mathematical Educator. 2003, Vol. 13, No. 2,47-57.

Jirotkova, D., Littler, G. (2004). Insight into pupil's understanding of infinity in a geometrical context. Proceedings of the 28thconference of the international group for the psychology of mathematics education. Retrieved from: http://www.kurims.kyoto-u.ac.jp/EMIS/proceedings/PME28/RR/RR079_Jirotkova.pdf\

Kattou, M. (2010). Teachers' Perceptions about Infinity: A process or an Object. Retrieved from: ife.ens-lyon.fr/publications/edition-electronique/cerme6/wg10-09-kattou.pdf

Moru, E. K., (2006). Epistemological obstacles in coming to understand the limit concept at undergraduate level: a case of the national university of Lesotho.

Singer, M., Voica, C. (2003). Perception of infinity: Does it really help in problem solving? Retrieved from: math.unipa.it/~grim/21_project/21_brno03_Singer.pdf

Tall, D., Schwarzenberger, R. (1978). Conflicts in the Learning of Real Numbers and Limits. Published in Mathematics Teaching, 82, 44–49.

Tall, D. (1980). Mathematical Intuition, with Special Reference to Limiting Processes. Published in Proceedings of the Fourth International Conference for the Psychology of Mathematic Education Berkeley 170-176.

Tall, D. (1990). Inconsistencies in the learning of calculus and analysis. Retrieved from: homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1990b-inconsist-focus.pdf

Tall, D. (1999). Reflections of the APOS Theory in Elementary and Advanced Mathematical Thinking. Proceedings of the 23rdConference of PME, Haifa, Israel, 1, 111-118. Retrieved from: homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1999c-apos-in-amt-pme.pdf

Tall, D. (2001a). Natural and formal infinities. Retrieved from: http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2001p-esm-infinity.pdf

Tall, D. (2001b). A child thinking about Infinity. Mathematics Education Research Centre University of Warwick Retrieved from: http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2001l-childs-infinity.pdf