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Full Length Research Paper

The Symmetry of Matter and Antimatter

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ABSTRACT

A formal and empirical review of matter and antimatter symmetry in QFT motivates a single particle reference hypothesis which is matched against recent observable data.

Key words: matter, antimatter, particle, antiparticle, relativistic quantum mechanics, quantum field theory, stability, annihilation, *baryogenesis*, standard model

1. INTRODUCTION

Physics is defined by the basic concept of matter (*viz.* energy) and the interactions thereof. As such, physics is categorized as a natural, empirical science.
On a rather phenomenological level, these interactions are successfully described in a more or less complex framework of space and time with quantitative notions about motion and force.
However, up until today the basic concept of matter remains a mystery, if not *the mystery* yet to be deciphered.
While day-to-day reason tends to define matter as something that has *mass* and occupies *space* with the property of a *volume*, modern physics work with a totally different concept, deeply rooted in its very formalism initiated by *Paul Adrien Maurice Dirac* (1902-1984), with his formal discovery of antimatter in his famous equation of 1928 [1].
Antimatter is a direct consequence of the theory of relativity and of quantum mechanics, i.e., of the quantum field theories (QFT) which underlie modern particle physics. Its theoretical prediction and experimental manifestation [2] are considered as one of the great successes of physics.
For every particle there is an antiparticle with quantum numbers and charges of the opposite sign.
Whether considering the relativistic wave equation for the electron that has both, positive and negative energy solutions, or current field theories with creation or annihilation operators acting on a field that create particles, destroy antiparticles or destroy particles and create antiparticles associated with the field, the properties and dynamics of particles and antiparticles are symmetrically correlated.

2. FORMAL SYMMETRY

The absolute symmetry between matter and antimatter is best demonstrated by briefly recapitulating the formalism of the free *Dirac-equation*, i.e., without any electromagnetic action potential:

$$(1) \qquad i\hbar\partial\psi(\vec{r},t)/\partial t = -i\hbar c\sum_{r=1}^{3}\alpha_r\,\partial\psi(\vec{r},t)/\partial x_r + \beta mc^2\psi(\vec{r},t)$$

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Since the functions $\psi \vec{p}(\vec{r},t)$ are eigenvalues of the impulse operator:

$$(2) \qquad \psi \vec{p}(\vec{r},t) = (2\pi\hbar)^{-3/2} \psi e^{i/\hbar(\vec{p}\vec{r}-Et)}$$

we need to determine the spinor

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$ in a way to solve (1)

through the impulse operator (2). This yields the equation system for the energies E with $\vec{p} = (p_x, p_y, p_z)$ as

(3)

$$\begin{aligned}
(E-mc^2)\psi_1 - c(p_x - ip_y)\psi_4 - cp_z\psi_3 &= 0 \\
(E-mc^2)\psi_2 - c(p_x + ip_y)\psi_3 + cp_z\psi_4 &= 0 \\
(E+mc^2)\psi_3 - c(p_x - ip_y)\psi_2 - cp_z\psi_1 &= 0 \\
(E+mc^2)\psi_4 - c(p_x + ip_y)\psi_1 + cp_z\psi_2 &= 0
\end{aligned}$$

Now we choose the *z*-axis in the direction of the impulse $\vec{p}$ to get x and $y = 0$, i.e., $\vec{p} = (0,0,p)$ yields pairs to determine ψ_1 and ψ_3 ((4a) and (4c)) as well as ψ_2 and ψ_4 respectively ((4b) and (4d)):

$$\begin{aligned}
&(4a) \qquad (E-mc^2)\psi_1 - cp\psi_3 = 0 \\
&(4b) \qquad (E-mc^2)\psi_2 + cp\psi_4 = 0 \\
&(4c) \qquad (E+mc^2)\psi_3 - cp\psi_1 = 0 \\
&(4d) \qquad (E+mc^2)\psi_4 + cp\psi_2 = 0
\end{aligned}$$

With $E^2 = m^2c^4 + c^2p^2$ we have the relativistic expression for the energy of a *single*, free particle where the energy E represents *as well as both*, positive and negative values for the same impulse $\vec{p}$ of a single particle:

$$(5) \qquad \begin{aligned} E_+ &= +\sqrt{c^2p^2 + m^2c^4} \\ E_- &= -\sqrt{c^2p^2 + m^2c^4} \end{aligned}$$

Figure 1 illustrates the theoretical energy spectrum of (5) giving rise to a fundamental problem with the very existence of matter, hence, with the existence of *everything* at stake:

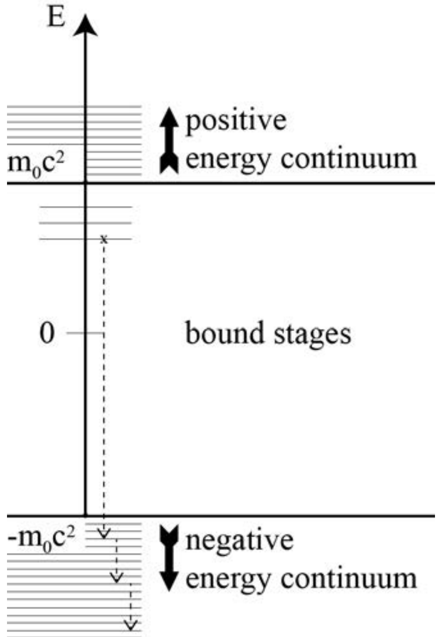

Figure 1: radiation catastrophe - an electron bound in an atom "falls" into the negative energy continuum with continuous emission of radiation.

How can matter exist? Taking the formal symmetry between matter and antimatter without any semantic reference hypothesis would physically imply atoms to be instable on principle. Matter would totally annihilate emitting pure energy in terms of light.

Obviously, just that is not observed. Nevertheless, considering

$$(\underline{5}) \qquad E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$

the fundamental problem of absolute symmetry of all points of matter (i.e., m_0) exists as well as both, on principle and in practice since the physical world exists.
The commonly accepted workaround is based on a semantic reference hypothesis originally proposed by *Dirac* himself [3].

For the purpose of providing an insight about the developed creativity to avoid negative energy properties, the general solutions for plane waves of the free *Dirac-equation* (6a and 6b) are further considered (cf. [4]):

$$(6a)\quad \psi(\vec{x},t)=\sum_{\pm s}\int d^3\vec{p}h^{-3/2}\sqrt{\frac{m_0c^2}{E_p}}\left[b(\vec{p},s)u(\vec{p},s)e^{(i/\hbar)(\vec{p}\vec{x}-E_pt)}+d^+(\vec{p},s)v(\vec{p},s)e^{(-i/\hbar)(\vec{p}\vec{x}-E_pt)}\right]$$

$$(6b)\quad \psi^+(\vec{x},t)=\sum_{\pm s}\int d^3\vec{p}h^{-3/2}\sqrt{\frac{m_0c^2}{E_p}}\left[b^+(\vec{p},s)\bar{u}(\vec{p},s)\gamma_0e^{(-i/\hbar)(\vec{p}\vec{x}-E_pt)}+d(\vec{p},s)\bar{v}(\vec{p},s)\gamma_0e^{(i/\hbar)(\vec{p}\vec{x}-E_pt)}\right].$$

The spinors $u(\vec{p},s),v(\vec{p},s)$ satisfy the *Dirac-equation* and constitute a complete orthogonal system.

$\psi^+(\vec{x},t)$ is hermitically conjoint to $\psi(\vec{x},t)$ where $b(\vec{p},s)$, $d(\vec{p},s)$, $b^+(\vec{p},s)$ and $d^+(\vec{p},s)$ are development-coefficients.

$E_p=c\sqrt{\vec{p}^2+m_0^2c^2}$. γ_0 is a constant matrix. The index s summarizes the spin-states.
The 2nd quantization of the *Dirac-field* now turns the development-coefficients b, d, b^+, d^+ into operators. The exchange-relations are chosen in a way that the *Pauli-principle* (exclusion principle) is satisfied. If the following anti-exchange relations apply $([A,B]_+=AB+BA)$, it guarantees formally that:

(7)

$$[b(\vec{p},s),b^+(\vec{p}\,',s')]_+=\delta_{ss'}\cdot\delta^3(\vec{p}-\vec{p}\,')$$
$$[d(\vec{p},s),d^+(\vec{p}\,',s')]_+=\delta_{ss'}\cdot\delta^3(\vec{p}-\vec{p}\,').$$

This yields to the anti-exchange relations for the fields

$$[\psi(\vec{x},t),\ \psi(\vec{x}\,',t)]_+=0$$
$$[\psi^+(\vec{x},t),\ \psi^+(\vec{x}\,',t)]_+=0.$$

In analogy to common procedures in QFT, (6b) b^+ and d are interpreted to be generation-operators for electrons. But now d generates a state with negative energy. In order to realize that, the energy-operator H (8) is expressed by b, d, b^+, d^+.

$$(8)\quad H=\sum_{\pm s}\int d^3\vec{p}E_p\left(b^+(\vec{p},s)b(\vec{p},s)-d(\vec{p},s)d^+(\vec{p},s)\right).$$

Interpreting the operator b^+ as generating a particle with positive energy $(E_p,\vec{p})$ and transferring this interpretation formally to the second operator in (8) suggests the conclusion that d^+ generates a particle with negative energy $(-E_p,-\vec{p})$.

The complete impulse-operator reads

$$\vec{p}=\sum_{\pm s}\int d^3\vec{p}\vec{p}\left(b^+(\vec{p},s)b(\vec{p},s)-d(\vec{p},s)d^+(\vec{p},s)\right).$$

Equation (8) shows that the continuum-solutions of the energy expression are not positive definite. But applying *Dirac's* semantic reference hypothesis (Fig. 2) according to which the vacuum is defined by all negative energy states being fully occupied with anti-electrons (positrons) and the positive energy states being completely empty, the energy and impulse operators are just rewritten as:

$$H = \sum_{\pm s} \int d^3\vec{p} E_p \big(b^+(\vec{p},s) b(\vec{p},s)$$
$$+ d^+(\vec{p},s) d(\vec{p},s) - -[d(\vec{p},s), d^+(\vec{p},s')]_+\big)$$
$$\vec{p} = \sum_{\pm s} d^3\vec{p}\vec{p} \big(b^+(\vec{p},s) b(\vec{p},s) + d^+(\vec{p},s) d^+(\vec{p},s) -$$
$$-[d(\vec{p},s), d^+(\vec{p},s')]_+\big).$$

In anticipation of the desired creation- and annihilation operators, the energy is kept positive. For a vacuum, the first two terms cancel because there are neither positive energy states which could annihilate with $b(\vec{p},s)$ nor states of negative energy which could be filled by $d(\vec{p},s)$. The last term is supposedly just an infinite constant (the total negative energy of the vacuum) which will proof critical in furtherance of this presentation. For now, it is simply ignored (normalized).
Thus, an anti-electron (positron) is just a (positive) manifestation of a (negative) hole in the *Dirac-sea* (Figure 2):

3. EMPIRICAL SYMMETRY

The formally discussed, Lorentz-invariant QFT equations are the foundations of the "Standard Model" (SM) of particle physics. If we flip the signs of all charges we turn particles into antiparticles. If we perform a consecutive space reversal $\vec{x} \to -\vec{x}$ and then a time reversal $t \to -t$, we recover the original equations.
This symmetry is called "CPT" (Charge-Parity-Time reversal) symmetry. It fundamentally implies that particles and antiparticles have exactly the same mass and that antimatter is an exact mirror of matter with all physical phenomena so far observed being invariant under CPT conjugation.

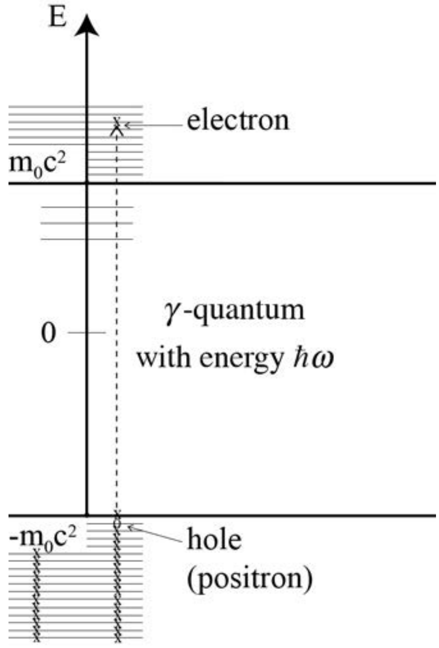

Figure 2: A photon with the energy $\hbar\omega \geq 2m_0c^2$ creates an electron-electron-hole-sate where the "hole" is interpreted as a positive electron (positron).

Hence, hydrogen atoms and their corresponding anti-hydrogen atoms should have exactly the same energy levels.
And just that has been empirically demonstrated at CERN [5].
Taking a look with the SM at the universe as a whole, matter is continuously transformed, e.g., in stars. Nuclear transformations such as the reaction *proton → neutron + positron + neutrino* change certain properties where, e.g., the number of protons decrease while neutrons increase.
Nevertheless, all empirical data gathered and analyzed so far strongly suggests that the more general class of particles called *baryons*, which includes protons and

neutrons, as well as *leptons* such as positrons and neutrinos, don't change but *remain absolute constant*.

4. GEDANKENEXPERIMENTS

To sum-up the formal and empirical situation of the well-tested SM, we set up simple *Gedankenexperiments*:

a) We imagine a dedicated region with absolutely no baryons nor leptons, neither matter nor antimatter, i.e., a total baryon and lepton number of zero.

Now we intend to introduce matter by interactions of high-energetic particles as actually done in particle accelerators and assumed by the SM in the early stage of the universe.

Since these interactions conserve the baryon and lepton number, they always result in pairs of baryons and antibaryons as well as leptons and antileptons created.

If we consider in line with the SM that baryons (B+) and antibaryons (B-) as well as leptons (L) and antileptons (L-) nullify each other by (almost) instant decay (B+/B- and L+/L- annihilation), the net number of B and L in our dedicated region would remain zero.

b) Now we imagine the opposite, i.e., a region with an equal amount of B+/B- and L+/L- where we intend to remove, e.g., the antimatter (B-/L-).

Since the SM prohibits spontaneous decay into particles *with less* net baryonic/leptonic content, the only process observable would be a B+/B- and L+/L- annihilation yielding again no net change in baryon and lepton number.

5. MATTER MYSTERY

Mysteriously, the so far presented properties of matter and antimatter seem to proof formally and empirically that the world of matter won't exist. This is why mainstream physics is still in the process of trying to decipher the relations of matter and antimatter by postulating an asymmetry resulting in an abundance of one over the other.

But the unsolved problems by assuming the existence of just a single region of our universe with a significant baryon number $\neq 0$ yield even more mysteries (cf. [6]), with two major implications:

As already laid out, standard particle physics only knows of formal and empirical processes which increase or decrease baryons and leptons *in pairs*, i.e., we don't know nor did we ever produce a change in the net number of B or L.

If now the inferred B/L > 0 (or just B > 0) shall be the case to have some starting point to physically explain the stability, hence, existence of matter as perceived, B/L > 0 must represent an initial condition of the SM.

Those however, who follow the formal and empirical symmetry of the universe with the universe having B/L = 0, thus equal amounts of matter and antimatter, postulate a spatial mechanism that needs to rearrange and separate matter from antimatter to treat it as distinct spatial domains.

A part of several attempts to formalize such domain separations [7], no consistent mechanism to prevent matter and antimatter domain interactions which would otherwise annihilate by either vanishing completely or lighting up the universe with annihilation gamma rays is known to-date.

Another alternative, although radical, was proposed by *Andrei Dmitrievich Sakharov* (1921-1989) [8] who just allows for baryon number violation declaring the SM as wrong at higher energies than currently available for empirical testing with accelerators. Unlike CPT symmetry, which follows from above discussed relativistic QFT and their implied Lorentz-invariance, baryon number variation is open to a variety of theoretical debate.

Sakharov himself set three conditions which could lead to a *baryogenesis*, i.e., to a dynamical generation of a baryon asymmetry from an initially symmetric universe:

1. The possibility of baryon number violation
2. CP symmetry violation
3. An initial thermal equilibrium

But because the underlying physics for *baryogenesis* is still completely unknown, there is a lot of speculation as to what spatial distribution of matter and antimatter is actually produced by *baryogenesis* (cf. [9]).

6. EPISTEMOLOGY

The description of *particles* with the language of physics, i.e., mathematics, constitutes both, its expressive power as well as its limits.

While the formalism of Lorentz-invariant QFT equations yields mathematical solutions which express more than their non-relativistic, non-quantum mechanical counterparts, i.e., positive *and* negative energy/mass properties, they also impose new limits with regard to their area of application.

Reviewing, e.g., the very mechanism of anticipating relativistic particles within QFT, the concept of a free particle is revised and limited by quantum mechanical measurement restrictions (*Heisenberg's Uncertainty principle*):

Because $\Delta x \sim \frac{\hbar}{\Delta p} \sim \frac{\hbar}{2m_0c}$ limits our measurement process on principle to a wavelength (*Compton wavelength*) of $\lambda_c = \frac{\hbar}{m_0c}$, any particle localization below λ_c yields to particle-antiparticle pair-creation with energies $> 2m_0c^2$.

Accordingly, the concept of a single particle is absolutely limited to localizations above the *Compton wavelength* so that elementary particles such as electrons, quarks, or photons, and composite particles such as protons or neutrons have *no spatial localization.* And just as with the non-spatial localization of particles the same restriction is imposed on their localization in *time* where $\Delta x > \frac{\hbar}{2m_0c}$ yields $\Delta t \sim \frac{\Delta x}{\Delta c} > \frac{\hbar}{2m_0c}$ so that particles are rather described in probabilistic terms ϱ to localize them in (x, y, z, t).

6.1 Single particle hypothesis

Looking at free particles as entities with no spatial and temporal localization provides with evidence to investigate the possibility of a single particle reference hypothesis yet not considered:

Particle and antiparticle have exactly the same localization, i.e., are just one entity without any interaction.

Once such a particle (*bi-particle*) is localized below $\lambda_c = \frac{\hbar}{m_0c}$ with $E \geq 2m_0c^2$, the well-established QFT interactions between particles and antiparticles are observed, including but not limited to particle-antiparticle annihilation, i.e.:

Every particle above $\lambda_c = \frac{\hbar}{m_0c}$ is a stable *bi-particle*.

6.2 Epistemological positioning

With this reference hypothesis being subject to our measurement restrictions imposed by quantum mechanical measurement properties, it essentially shifts the existential constraint of the

stability with a mere phenomenological reference hypothesis to an ontological one:
Instead of trying to alter a consistent and empirically successful formalism with well-understood and tested pair-creation and annihilation processes for presumably securing a physical explanation for the stability of matter, it leaves the valid question about stability and existence of matter for an ontological rather than phenomenological debate.

7. CONCLUSION

While there is widespread belief that the negative energies were once and for all understood in terms of antiparticles, they are not related to non-localized, non-temporal negative energy states whose quanta are by construction in QFT neither created nor annihilated above a localization of $\lambda_c = \frac{\hbar}{m_0 c}$, i.e., below energies of $2m_0c^2$.

As previously presented, the actual meaning of field negative frequency terms in QFT does not bypass the negative energies since the corresponding solutions were neglected from the beginning with phenomenological *ad hoc* reference hypothesis as initiated by *Dirac* himself.
The drawback for attenuating or totally neglecting negative energy solutions implies all those field vacuum divergences that formally arise after quantization along with all initiatives to cancel such infinities without reintroducing negative energy states (vacuum energy).
On a heuristic vein, the SM doesn't require any *ad hoc* modification in terms of *baryogenesis* nor are domain separation mechanisms required to prevent particle-antiparticle annihilation.
With cosmological observations of supernovae, WMAP [10] and COBE [11] satellite's data on Cosmic Microwave Background (CMB) as well as galaxy cluster distributions, the observational evidence is growing that our universe is presently in a state of accelerated expansion.
The majority of researchers is accounting for this empirical data with the reintroduction of a cosmological constant, *dark matter* with some kind of negative pressure, or with scalar fields with negative kinetic energy, i.e., *phantom fields* (cf. [12] [13] [14]).
But because all of these models are built on QFT they unavoidably yield a violation of positive energy conditions and are constraint by quantum instability of the vacuum where the common *trick* is to impose an *ultraviolet cutoff* to effectively restrict the theory to low energy and to keep the instability at an unobservable rate.
However, while stability is the main concern for any physical model trying to incorporate negative energy fields, it is only the *interaction* with positive energy fields which causes persistent theoretical failure.
The here presented, symmetric, bi-particle hypothesis may provide new insights to gravitational interaction and may derive rich phenomenological and theoretical perspectives.

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